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Potential Vorticity (PV) Dynamics and Models of Zonal Flow Formation

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APS-DPP, Nov 16-20, 2015

Outline

I. Introduction

II. Non-perturbative MFT of turbulent relaxation and PV transport

– structure of PV flux (1)(2)

i. Selective decay model: minimum enstrophy principle

ii. PV-avalanche model: symmetry principles

II. Perturbation theory of PV flux

– transport coefficients (1)

i. Modulational instability

ii. Parametric instability

III. Zonal flow formation in the presence of ambient mean shear (3)

IV. Summary

(1) Pei-Chun Hsu and P. H. Diamond, Phys. Plasmas, 22, 032314 (2015)

(2) Pei-Chun Hsu, P. H. Diamond, and S. M. Tobias, Phys. Rev. E, 91, 053024 (2015)

(3) Pei-Chun Hsu and P. H. Diamond, Phys. Plasmas, 22, 022306, (2015)

PV mixing & ZF formation

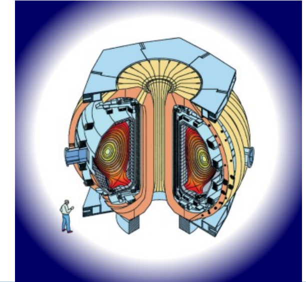
- PV conservation $\frac{dq}{dt} = 0$

GFD: Quasi-geostrophic system	Plasma: Hasegawa-Wakatani system
$q = \nabla^2\psi + \beta y$	$q = n - \nabla^2\phi$
$\nabla^2\psi$: relative vorticity; βy: planetary vorticity	n: density (guiding center); $\nabla^2\phi$: ion vorticity (polarization)
Physics: $\Delta y \rightarrow \Delta\left(\nabla^2\psi\right) \rightarrow$ ZF!	Physics: $\Delta r \rightarrow \Delta n \rightarrow \Delta\left(\nabla^2\phi\right) \rightarrow$ ZF!

- Zonal Flow formation

Taylor's identity $\left\langle \tilde{v}_y \tilde{q} \right\rangle = -\frac{\partial}{\partial y}\left\langle \tilde{v}_x \tilde{v}_y \right\rangle$ → **PV flux fundamental to ZF formation**

Introduction:
physical systems



turbulence	quasi-geostrophic	drift-wave
→ force	Coriolis	Lorentz
velocity	geostrophic	magnetic energy
linear waves	Rossby waves	drift waves
→ conserved PV	$q = \nabla^2 \psi + \beta y$	$q = n - \nabla^2 \phi$
→ inhomogeneity	β	$\nabla n, \ \nabla T$
→ characteristic scale	$L_D \approx 10^6 m$	$\rho_s \approx 10^{-3} m$
→ fast frequency	$f \approx 10^{-2} s^{-1}$	$\omega_{ci} \approx 10^8 s^{-1}$
turbulence	usually strongly driven	not far from marginal
Reynolds number	R_e >>1	R_e ~ 10-10^2
→ zonal flows	Jets, zonal bands	sheared E x B flows
role of zonal flows	transport barriers	L-H transition

Non-perturbative approaches

- PV mixing in space is essential in ZF generation.

Taylor identity: $\left\langle \tilde{v}_y \nabla^2 \tilde{\phi} \right\rangle = -\partial_y \left\langle \tilde{v}_y \tilde{v}_x \right\rangle$

vorticity flux   Reynolds force

Key:
How represent inhomogeneous PV mixing

→

General structure of PV flux?
→relaxation principles!

most treatment of ZF:
-- perturbation theory
-- modulational instability
(test shear + gas of waves)
~ linear theory

-> physics of evolved PV mixing?
-> something more general?

non-perturb model 1: use selective decay principle

What form must the PV flux have so as to dissipate enstrophy while conserving energy?

non-perturb model 2: use joint reflection symmetry

What form must the PV flux have so as to satisfy the joint reflection symmetry principle for PV transport/mixing?

non-perturb model 1

General principle: selective decay

- 2D turbulence conservation of energy and potential enstrophy

→ dual cascade

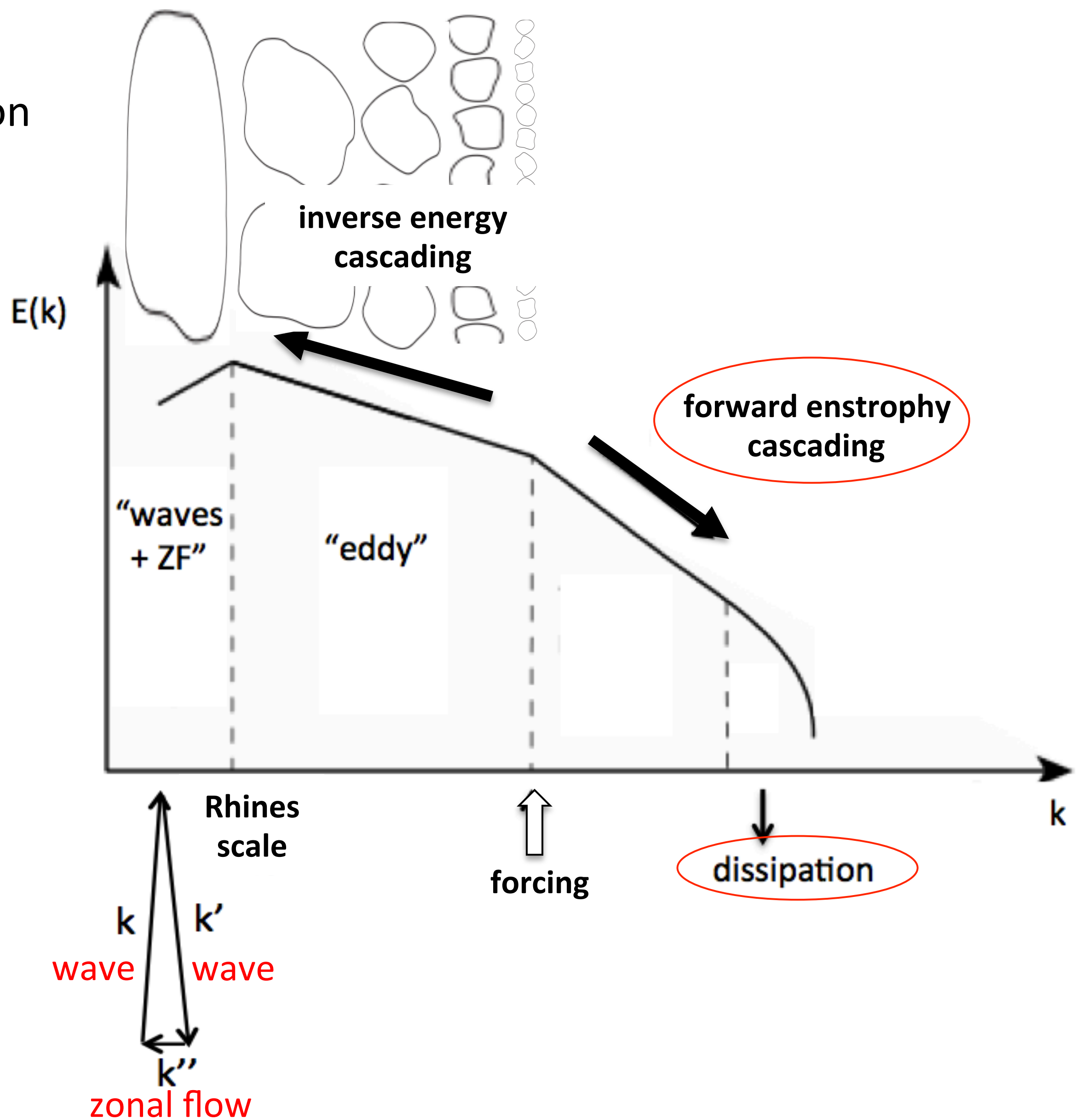
→ Minimum enstrophy state

- eddy turnover rate and Rossby wave frequency mismatch are comparable

$$\frac{\partial \omega}{\partial t} + \vec{u} \cdot \nabla \omega + \beta v = 0$$

$$\frac{U}{LT} \quad \frac{U^2}{L^2} \quad \beta U$$

→ Rhines scale $L_R \sim \sqrt{\frac{U}{\beta}}$



non-perturb model 1

Using selective decay for flux

	minimum enstrophy relaxation (Bretherton & Haidvogel 1976)	analogy ↔	Taylor relaxation (J.B. Taylor, 1974)
turbulence	2D hydro		3D MHD
conserved quantity (constraint)	total kinetic energy		global magnetic helicity
dissipated quantity (minimized)	fluctuation potential enstrophy		magnetic energy
final state	minimum enstrophy state flow structure emergent		Taylor state force free B field configuration
structural approach	$\frac{\partial}{\partial t}\Omega < 0 \Rightarrow \Gamma_E \Rightarrow \Gamma_q$		$\frac{\partial}{\partial t}E_M < 0 \Rightarrow \Gamma_H$

dual cascade (conserved quantity / dissipated quantity)

- flux? what can be said about dynamics?

→ structural approach (this work): *What form must the PV flux have so as to dissipate enstrophy while conserving energy?*

General principle based on general physical ideas → useful for dynamical model

non-perturb model 1

PV flux

→ PV conservation

mean field PV: $$\frac{\partial \langle q \rangle}{\partial t} + \partial_y \underbrace{\langle v_y q \rangle}_{} = \nu_0 \partial_y^2 \langle q \rangle$$

Γ_q : mean field PV flux

Key Point:
form of PV flux Γ_q which dissipates enstrophy & conserves energy

selective decay

→ energy conserved $E = \int \frac{\left(\partial_y \langle \phi \rangle\right)^2}{2}$

$$\frac{\partial E}{\partial t} = \int \langle \phi \rangle \partial_y \Gamma_q = -\int \partial_y \langle \phi \rangle \Gamma_q \quad \Rightarrow \Gamma_q = \frac{\partial_y \Gamma_E}{\partial_y \langle \phi \rangle}$$

→ enstrophy minimized $\Omega = \int \frac{\langle q \rangle^2}{2}$

$$\frac{\partial \Omega}{\partial t} = -\int \langle q \rangle \partial_y \Gamma_q = -\int \partial_y \left(\frac{\partial_y \langle q \rangle}{\partial_y \langle \phi \rangle} \right) \Gamma_E$$

$$\frac{\partial \Omega}{\partial t} < 0 \Rightarrow \Gamma_E = \mu \partial_y \left(\frac{\partial_y \langle q \rangle}{\partial_y \langle \phi \rangle} \right)$$

μ ↓ parameter TBD; $\partial_y \langle \phi \rangle$ ↘ $\langle v_x \rangle$

$$\Rightarrow \Gamma_q = \frac{1}{\partial_y \langle \phi \rangle} \partial_y \left[\mu \partial_y \left(\frac{\partial_y \langle q \rangle}{\partial_y \langle \phi \rangle} \right) \right]$$

general form of PV flux

non-perturb model 1

Structure of PV flux

$$\Gamma_q = \frac{1}{\langle v_x \rangle} \partial_y \left[\mu \, \partial_y \left(\frac{\partial_y \langle q \rangle}{\langle v_x \rangle} \right) \right] = \frac{1}{\langle v_x \rangle} \partial_y \left[\mu \left(\frac{\langle q \rangle \partial_y \langle q \rangle}{\langle v_x \rangle^2} + \frac{\partial_y^2 \langle q \rangle}{\langle v_x \rangle} \right) \right]$$

diffusion parameter calculated by perturbation theory, numerics…

diffusion and hyper diffusion of PV

<--> usual story : Fick’s diffusion

relaxed state:

Homogenization of $\frac{\partial_y \langle q \rangle}{\langle v_x \rangle}$ → allows staircase

characteristic scale $\ell_c \equiv \sqrt{\left| \frac{\langle v_x \rangle}{\partial_y \langle q \rangle} \right|}$

$\ell > \ell_c$: zonal flow growth

$\ell < \ell_c$: zonal flow damping (hyper viscosity-dominated)

Rhines scale $L_R \sim \sqrt{\frac{U}{\beta}}$

$\ell > L_R$: wave-dominated

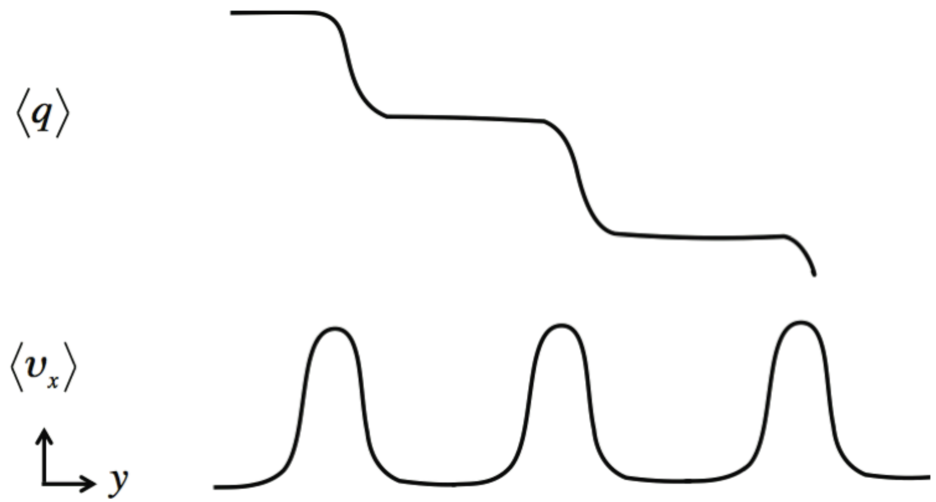
$\ell < L_R$: eddy-dominated

non-perturb model 1

PV staircase

relaxed state: homogenization of $\dfrac{\partial_y \langle q \rangle}{\langle v_x \rangle}$ → PV gradient large where zonal flow large

→ Zonal flows track the PV gradient → PV staircase



- Highly structured profile of the staircase is reconciled with the homogenization or mixing process required to produce it.
- Staircase may arise naturally as a consequence of minimum enstrophy relaxation.

What sets the "minimum enstrophy"

- Decay drives relaxation. The relaxation rate can be derived by linear perturbation theory about the minimum enstrophy state

$$\langle q \rangle = q_m(y) + \delta q(y,t)$$

$$\langle \phi \rangle = \phi_m(y) + \delta\phi(y,t)$$

$$\partial_y q_m = \lambda \partial_y \phi_m$$

$$\delta q(y,t) = \delta q_0 \exp(-\gamma_{rel} t - i\omega t + iky)$$

γ_{rel} >0 relaxation

$$\gamma_{rel} = \mu\left(\frac{k^4 + 4\lambda k^2 + 3\lambda^2}{\langle v_x \rangle^2} - \frac{8q_m^2(k^2+\lambda)}{\langle v_x \rangle^4}\right)$$

$$\omega_k = \mu\left(-\frac{4q_m k^3 + 10 q_m k\lambda}{\langle v_x \rangle^3} - \frac{8q_m^3 k}{\langle v_x \rangle^5}\right)$$

- The condition of relaxation (modes are damped):

$$\gamma_{rel} > 0 \;\Rightarrow\; k^2 > \frac{8q_m^2}{\langle v_x \rangle^2} - 3\lambda \;\Rightarrow\; \boxed{\frac{8q_m^2}{\langle v_x \rangle^2} > 3\lambda}$$

→ Relates q_m^2 with ZF and scale factor

$$\Rightarrow \langle v_x \rangle^2 < \frac{3\lambda}{8q_m^2}$$

ZF can't grow arbitrarily large

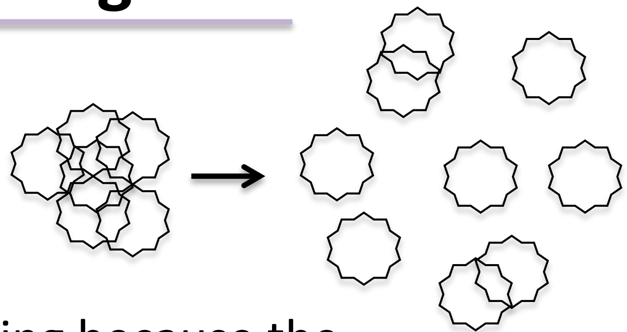
$$\Rightarrow 8\, q_m^2 > \langle v_x \rangle^2\, 3\lambda$$

the 'minimum enstrophy' of relaxation, related to scale

non-perturb model 1

Role of turbulence spreading

- Turbulence spreading: tendency of turbulence to self-scatter and entrain stable regime



- Turbulence spreading is closely related to PV mixing because the transport/mixing of turbulence intensity has influence on Reynolds stresses and so on flow dynamics.

- PV mixing is related to turbulence spreading

$$\frac{\partial E}{\partial t} = \int \langle \phi \rangle \partial_y \Gamma_q = -\int \partial_y \langle \phi \rangle \Gamma_q \qquad \Rightarrow \Gamma_q = \frac{\partial_y \Gamma_E}{\partial_y \langle \phi \rangle}$$

- The effective spreading flux of turbulence kinetic energy

$$\Gamma_E = -\int \Gamma_q \langle v_x \rangle dy = -\int \frac{1}{\langle v_x \rangle} \partial_y \left[\mu \partial_y \left(\frac{\partial_y \langle q \rangle}{\langle v_x \rangle} \right) \right] \langle v_x \rangle dy = \mu \partial_y \left(\frac{\partial_y \langle q \rangle}{\langle v_x \rangle} \right)$$

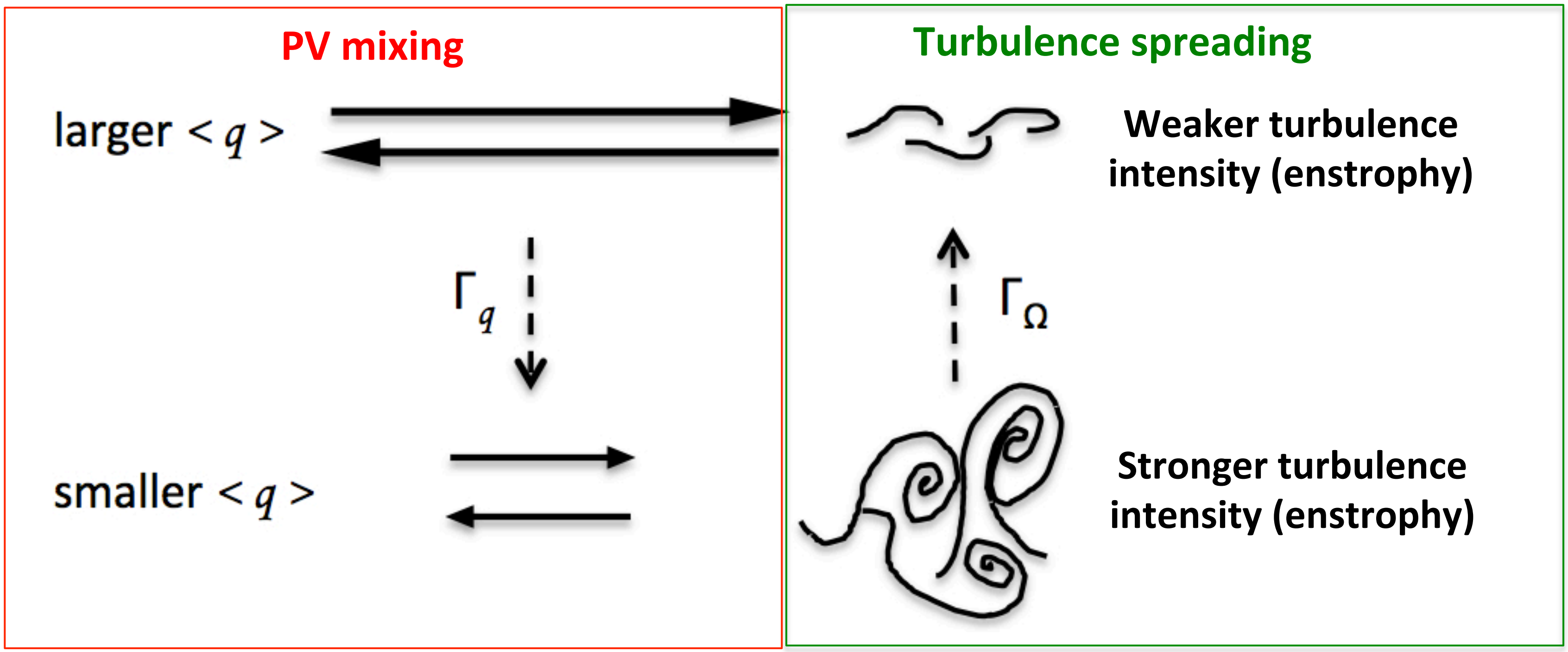
→the gradient of the $\partial_y \langle q \rangle / \langle v_x \rangle$, drives spreading

→ the spreading flux vanishes when $\partial_y \langle q \rangle / \langle v_x \rangle$ is homogenized

Discussion

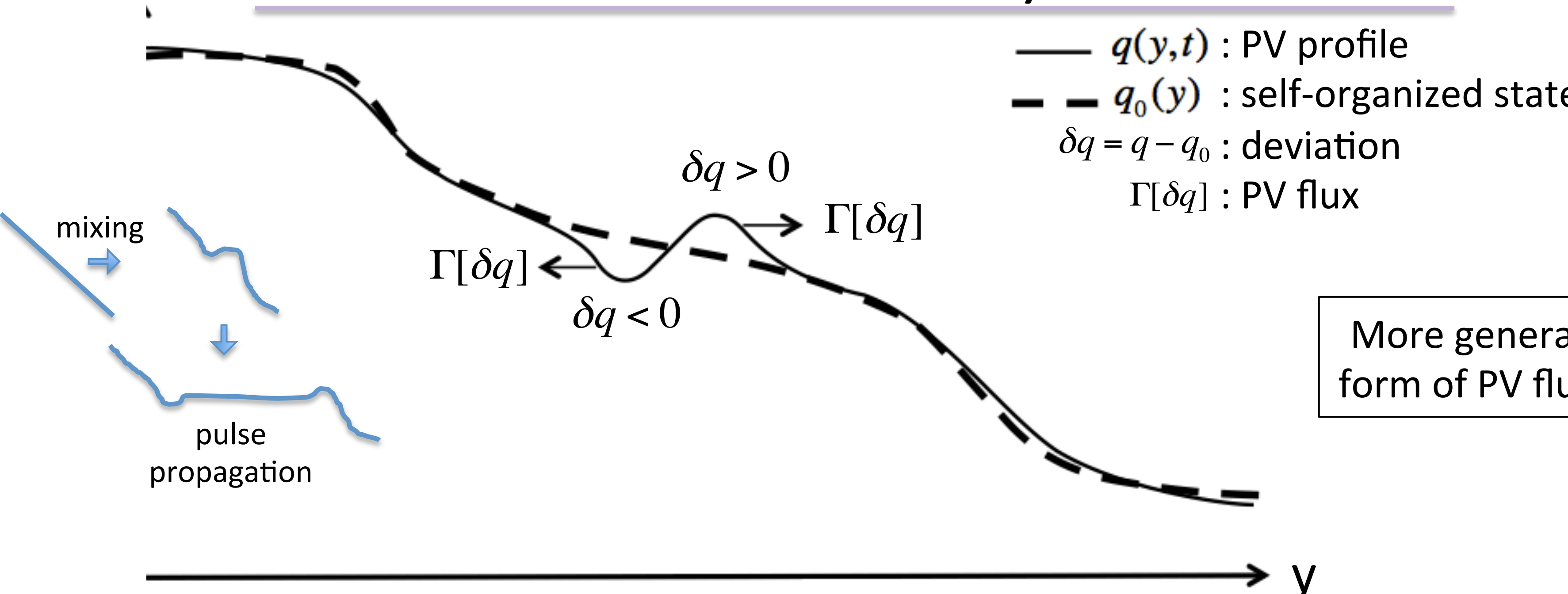
- PV mixing ↔ forward enstrophy cascade ↔ hyper-viscosity
  → How to reconcile effective negative viscosity with the picture of diffusive mixing of PV in real space?

- A possible explanation of up-gradient transport of PV due to turbulence spreading



non-perturb model 2

PV-avalanche model – beyond diffusion



More general form of PV flux

- avalanching: tendency of excitation to propagate in space via local gradient change
- Joint-reflection symmetry: Γ [δq] invariant under y → -y and δq → -δq

Key Point:
form of PV flux which satisfies joint-reflection symmetry

→ $\Gamma[\delta q] = \sum_l \alpha_l (\delta q)^{2l} + \sum_m \beta_m (\partial_y \delta q)^m + \sum_n \gamma_n (\partial_y^3 \delta q)^n + \ldots$

- large-scale properties : higher-order derivatives neglected
  small deviations : higher-order terms in δq neglected

→ Simplest approximation: $\Gamma[\delta q] = \frac{\alpha}{2}(\delta q)^2 + \beta \partial_y \delta q + \gamma \partial_y^3 \delta q$ general form of PV flux

non-perturb model 2

$$\Gamma[\delta q] = \frac{\alpha}{2}(\delta q)^2 + \beta\partial_y \delta q + \gamma\partial_y^3 \delta q$$

- PV equation: $$\partial_t \delta q + \alpha\delta q \partial_y \delta q + \beta\partial_y^2 \delta q + \gamma\partial_y^4 \delta q = 0$$

Kuramoto-Sivashinsky type equation

diffusion and hyper diffusion of δq

Non-linear convection of δq

- Avalanche-like transport is triggered by deviation of PV gradient
  - → PV deviation implicitly related to the local PV gradient $\delta q \to \partial_y q$
  - → transport coefficients (functions of δq) related to the gradient $D(\delta q) \to D(\partial_y q)$
  - → gradient-dependent effective diffusion $\Gamma_q \sim -D(\partial_y q)\partial_y q \to -D(\delta q)\delta q$

→ Convective component of the PV flux can be related to a gradient-dependent effective diffusion

$$\Gamma[\delta q] \sim \delta q^2 \to -D(\delta q)\delta q$$

$$D(\delta q) \to D_0 \delta q$$

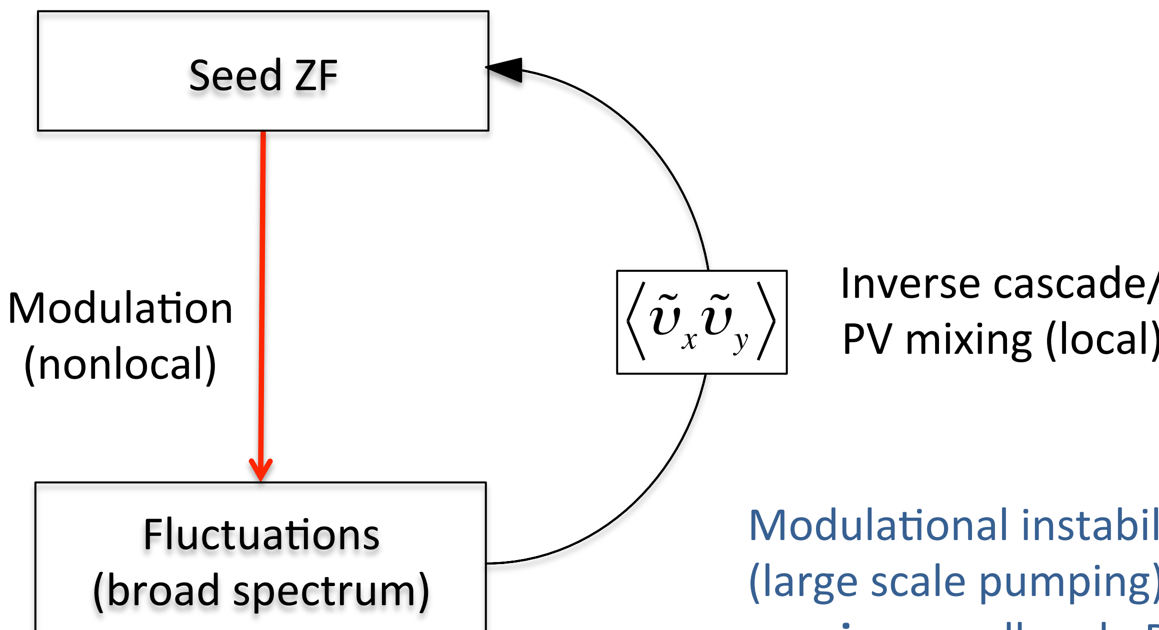
Perturbation theory

General structure of PV flux
→ relaxation principles

→

Transport coefficients of PV flux
→ perturbation theory
(only analytical solution)

- The evolution of perturbation (seed ZF) as a way to look at PV transport



Modulational instability
(large scale pumping)
requires small scale PV mixing

perturbation theory 1 – broadband

Revisiting modulational instability

ZF evolution determined by Reynolds force

$$\frac{\partial}{\partial t}\delta V_x = -\frac{\partial}{\partial y}\langle\tilde{\upsilon}_x\tilde{\upsilon}_y\rangle = \frac{\partial}{\partial y}\sum_{k<}\frac{k_x k_y}{k^4}\tilde{N}_k$$

vorticity flux

$N_k = k^2|\psi_k|^2/\omega_k$ is wave action density, for Rossby wave and drift wave, it is proportional to the enstrophy density. N_k is determined by WKE:

$$\frac{\partial\tilde{N}}{\partial t} + \upsilon_g\cdot\nabla\tilde{N} + \delta\omega_k\tilde{N} = \frac{\partial(k_x\delta V_x)}{\partial y}\frac{\partial N_0}{\partial k_y}$$

➜ Turbulent vorticity flux derived

$$\frac{\partial}{\partial t}\delta V_q = \partial_y^2\delta V_q\sum_k\left(\frac{k_x^2 k_y}{k^4}\right)\frac{\delta\omega_k}{(\omega_q - q\cdot\upsilon_g)^2 + \delta\omega_k^2}\frac{\partial N_0}{\partial k_y}$$

$\kappa(q)$

q: ZF wavenumber

$$\frac{\partial}{\partial t}\delta V_q = \partial_y^2\kappa(q)\ \delta V_q$$

$\kappa(q) \neq$ const

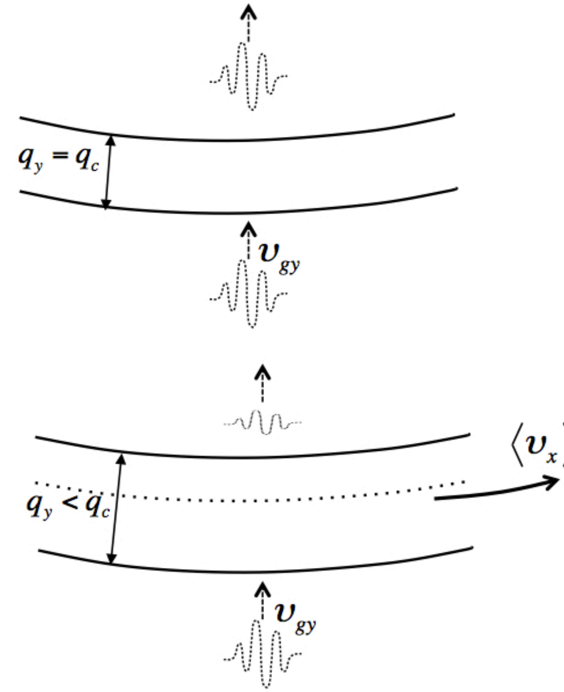
➜ scale dependence of PV flux

➜ non-Fickian turbulent PV flux

perturbation theory 1

- A simple model from which to view $\kappa\,(q)$:
  - Defining MFP of wave packets as the critical scale $q_c^{-1} \equiv \upsilon_g \delta\omega_k^{-1}$
  - keeping next order term in expansion of response

$$q^{-1} >> q_c^{-1} \Rightarrow \frac{\delta\omega_k}{(q\upsilon_g)^2 + \delta\omega_k^2} \approx \frac{1}{\delta\omega_k}\left(1 - \frac{q^2}{q_c^2}\right)$$



➜ zonal growth evolution:

$$\partial_t \delta V_x = D\partial_y^2 \delta V_x - H\partial_y^4 \delta V_x$$

➜ negative viscosity and positive hyper-viscosity

$$D = \sum_k \frac{k_x^2}{\delta\omega_k k^4} \frac{k_y \partial N_0}{\partial k_y} < 0$$

$$H = -\sum_k q_c^{-2} \frac{k_x^2}{\delta\omega_k k^4} \frac{k_y \partial N_0}{\partial k_y} > 0$$

< 0

⟺ Transport coefficients (viscosity and hyper-viscosity) for relaxation models:

$$\frac{\partial\langle \upsilon_x \rangle}{\partial t} = \Gamma_q = \frac{1}{\partial_y \langle \phi \rangle} \partial_y \left[\mu \left(-\frac{\langle q \rangle \partial_y \langle q \rangle}{\left(\partial_y \langle \phi \rangle\right)^2} + \frac{\partial_y^2 \langle q \rangle}{\partial_y \langle \phi \rangle} \right) \right]$$

$$\Gamma[\delta q] = \frac{\alpha}{2}(\delta q)^2 + \beta\partial_y \delta q + \gamma\partial_y^3 \delta q,$$

Discussion of *D* and *H*

- Roles of negative viscosity and positive hyper-viscosity **(Real space)**

$$\frac{\partial}{\partial t}\delta V_x = D\partial_y^2\delta V_x - H\partial_y^4\delta V_x$$

$$\frac{\partial}{\partial t}\int \frac{1}{2}\delta V_x^2 d^2x = -D\int\left(\partial_y\delta V_x\right)^2 d^2x - H\int\left(\partial_y^2\delta V_x\right)^2 d^2x$$

$D<0 \Rightarrow \gamma_{q,D}>0$ ZF growth (Pumper D)

$H>0 \Rightarrow \gamma_{q,H}<0$ ZF suppression (Damper H)

⇒ Energy transferred to large scale ZF

➜ D, H as model of spatial PV flux beyond over-simplified negative viscosity

$D = Hq^2$ sets the cut-off scale

$$\Rightarrow l_c^2 = \sqrt{\frac{H}{|D|}}$$

⇔

Minimum enstrophy model

$$\Gamma_q = \frac{1}{\partial_y\langle\phi\rangle}\partial_y\left[\mu\,\partial_y\left(\frac{\partial_y\langle q\rangle}{\partial_y\langle\phi\rangle}\right)\right] \Rightarrow \ell_c \equiv \sqrt{\left|\frac{\langle v_x\rangle}{\partial_y\langle q\rangle}\right|}$$

$\ell > \ell_c$: ZF energy growth → D process dominates at large scale

$\ell < \ell_c$: ZF energy damping → H process dominates at small scale

Parametric instability

	pseudo-fluid	plasma fluid
elements	wave-packets	charged particles (species α)
distribution function	$N_{\boldsymbol{k}}(\boldsymbol{k}, \omega_{\boldsymbol{k}})$	$f_\alpha(\boldsymbol{r}, \boldsymbol{v}, t)$
mean free path	$\lvert\boldsymbol{v}_g\rvert/\delta\omega_{\boldsymbol{k}}$	$1/n_\alpha \sigma$
density	$n^w = \int N_{\boldsymbol{k}} d\boldsymbol{k}$	$n_\alpha = \int f_\alpha d\boldsymbol{v}$
momentum	$\boldsymbol{P}^w = \int \boldsymbol{k} N_{\boldsymbol{k}} d\boldsymbol{k}$	$\boldsymbol{p}_\alpha = \int m_\alpha \boldsymbol{v} f_\alpha d\boldsymbol{v}$
velocity	$\boldsymbol{V}^w = \frac{\int \boldsymbol{v}_g N_{\boldsymbol{k}} d\boldsymbol{k}}{\int N_{\boldsymbol{k}} d\boldsymbol{k}}$	$\boldsymbol{u}_\alpha = \frac{\int \boldsymbol{v} f_\alpha d\boldsymbol{v}}{\int f_\alpha d\boldsymbol{v}} = \frac{\boldsymbol{p}_\alpha}{m_\alpha n_\alpha}$

Pei-Chun Hsu and P. H. Diamond, Phys. Plasmas, 22, 032314 (2015)

perturbation theory 2

- ZF evolution:

$$\langle \tilde{\upsilon}_x \tilde{\upsilon}_y \rangle = \int \upsilon_{gy} k_x N_k d^2k \cong V_y^w P_x^w$$ pseudo-momentum flux

$$\frac{\partial}{\partial t}\langle \upsilon_x \rangle = -\frac{\partial}{\partial y} V_y^w P_x^w$$

- pseudo-fluid evolution:

$$\frac{\int (WKE)\upsilon_{gy} d^2k}{n^w}$$

$$a = \int \frac{2\beta k_x^2}{k^4}\left(1 - \frac{4k_y^2}{k^2}\right) N_k dk \Big/ \int N_k \, dk$$

$$\Rightarrow \frac{\partial}{\partial t} V_y^w + V_y^w \frac{\partial}{\partial y} V_y^w = -a \langle \upsilon_x \rangle'$$

inviscid Burgers'eq.
source: zonal shear

→ ZF growth rate in monochromatic limit:

$$\gamma_q = \sqrt{q^2 k_x^2 |\varphi_k|^2 \left(1 - \frac{4k_y^2}{k^2}\right)}$$

-- Instability (γ_q real) → $k_x^2 > 3k_y^2$

-- $\gamma_q \propto |q|$ → convective instability

⟺ Convective transport coefficients for PV-avalanche model:

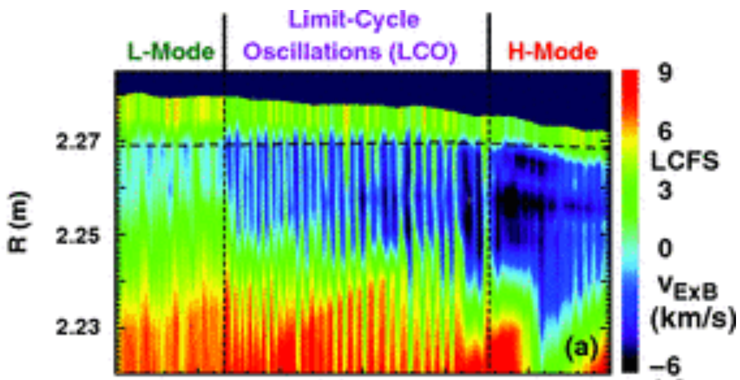
$$\Gamma[\delta q] = \frac{\alpha}{2}(\delta q)^2 + \beta \partial_y \delta q + \gamma \partial_y^3 \delta q,$$

	PV flux	convective	viscous	hyper-viscous	coefficients
(non-perturb.)	Min. enstrophy relaxation		•	•	
	PV-avalanche relaxation	•	•	•	
(perturbative)	Modulational instability		•	•	$D_t(<0), H_t(>0)$
	Parametric instability	•			$\gamma_q(\sim \lvert q \rvert)$

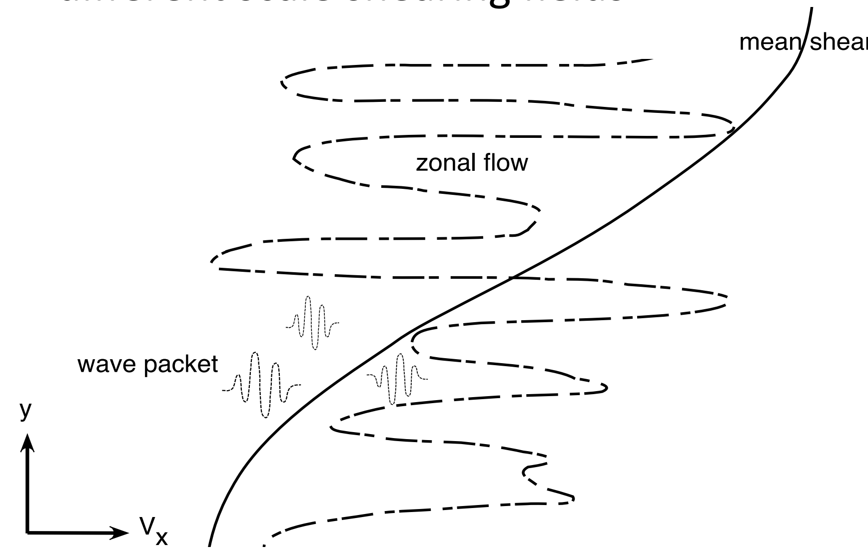
- Minimum enstrophy $$\frac{\partial \langle v_x \rangle}{\partial t} = \Gamma_q = \frac{1}{\partial_y \langle \phi \rangle} \partial_y \left[\mu \left(-\frac{\langle q \rangle \partial_y \langle q \rangle}{(\partial_y \langle \phi \rangle)^2} + \frac{\partial_y^2 \langle q \rangle}{\partial_y \langle \phi \rangle} \right) \right]$$
- PV-avalanche $$\Gamma[\delta q] = \frac{\alpha}{2} (\delta q)^2 + \beta \partial_y \delta q + \gamma \partial_y^3 \delta q$$
- Modulational instab. $$\partial_t \delta V_x = -q^2 D \delta V_x + q^4 H \delta V_x$$
- Parametric instab. $$\gamma_q = \sqrt{q^2 k_x^2 |\varphi_k|^2 \left(1 - \frac{4k_y^2}{k^2} \right)}$$

III) Multi-scale shearing effects

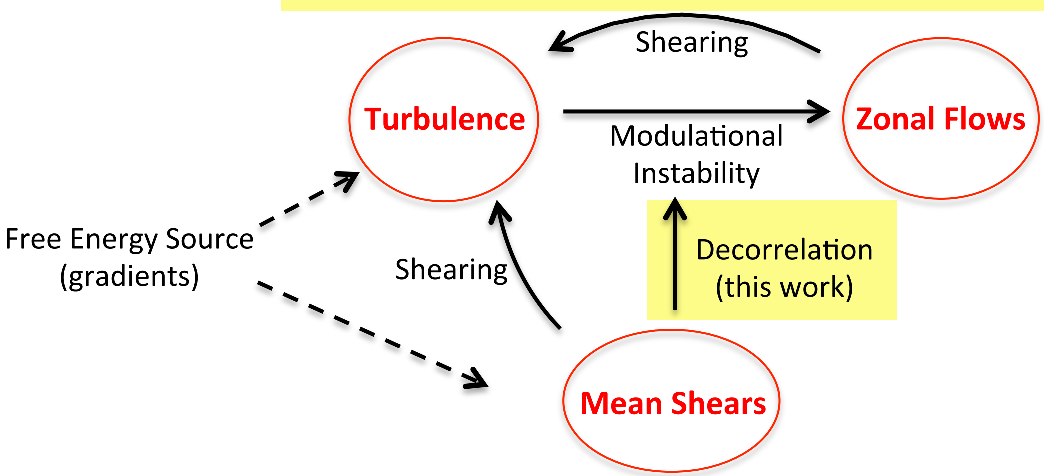
- Motivation: coexistence of mean shear and zonal flow shear (different roles)
  → L-H transition, the solar tachocline



- Generic problem: interaction between different scale shearing fields



- Important issue: how mean shear affects the PV flux and ZF generation



Pei-Chun Hsu and P. H. Diamond, Phys. Plasmas, 22, 022306, (2015)

Modulational instab. w/ mean shear

Momentum flux -> Reynolds stress -> wave action $<v_x v_y> = -\sum_k \frac{k_x k_y}{k^4} N_k$

- mean shear in WKE: $\partial_t \tilde{N}_k + v_{gy}\partial_y \tilde{N}_k - k_x \langle V_x \rangle' \partial_{k_y} \tilde{N}_k + \delta\omega_k \tilde{N}_k = k_x \delta V_x' \partial_{k_y} N_0$

  (shearing rate Ω)

  decrease $v_{gy} \propto \Omega^{-3}$

  mean shear $-\partial_y \langle V_x \rangle = \Omega$

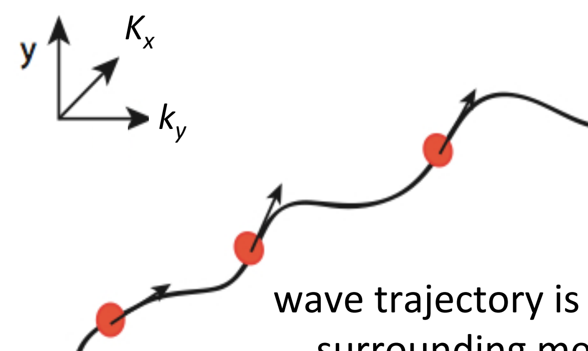
  Non-linear diffusion

  Seed ZF

- Ray trajectory refraction:

$$\frac{dk_y}{dt} = -\frac{\partial}{\partial y}(\omega + k_x V_x) \quad ; V_x = \langle V_x \rangle + \tilde{V}_x$$

$$\frac{dy}{dt} = v_{gy} = \frac{2\beta k_x k_y}{k^4}$$



wave trajectory is distorted by surrounding mean shears

$k_y(t) = k_y(0) + k_x \Omega t$ ↑ smaller scale

$y(t) = y(0) + e(t), \quad e(t) = \frac{\beta}{\Omega}\left(\frac{1}{k_0^2} - \frac{1}{k_x^2 + (k_{0y} + k_x \Omega t)^2}\right)$ ↓ excursion inhibited

Modulational instab. w/ mean shear

- characteristic method (shearing frame)

original frame $\left(\begin{array}{l} k_y = k_y(0) + k_x \Omega t \\ y = y(0) + e(t) \end{array} \right)$

$$\partial_t \tilde{N}_k + \underbrace{v_{gy} \partial_y \tilde{N}_k - k_x \langle V_x \rangle' \partial_{k_y} \tilde{N}_k}_{\text{cancel in shearing frame}} + \delta\omega_k \tilde{N}_k = k_x \delta V_x ' \partial_{k_y} N_0$$

cancel in shearing frame

shearing frame ( )

- Solving Green's function in shearing frame
- Changing variables back to original frame

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- strong mean shear limit ( ):

$$\gamma_q,\ \langle \tilde{v}_y \nabla^2 \tilde{\psi} \rangle \propto \left(\frac{3}{\delta\omega_k \Omega^2} \right)^{1/3}$$

→ Mean shear reduces ZF growth, PV flux ~$\Omega^{-2/3}$
→ scaling of PV flux in strong mean shear

Summary

- Inhomogeneous PV mixing is identified as the fundamental mechanism for ZF formation. *This study offered new approaches to calculating spatial flux of PV.*

- The general structure of PV flux is studied by two *non-perturbative* relaxation models. In selective decay model, PV flux contains diffusive and hyper-diffusive terms. In PV-avalanche model, PV flux contains another convective term, which can be generalized to an effective diffusive transport.

- The transport coefficients are derived using perturbation theory. In modulational instability analysis for a broadband spectrum, a negative viscosity and a positive hyper-viscosity, which represents ZF saturation mechanism, are derived. In parametric instability analysis for a narrow spectrum, a convective transport coefficient is obtained.

- Important issues addressed in our models includes *PV staircase, turbulence spreading, avalanche-like transport, characteristic scales*.

- The effect of the mean shear on ZF formation is studied. ZF growth rate and the PV flux are shown to decreases with mean shearing rate. *Framework of PV transport for systems with multi-scale shearing fields is established.*

Acknowledgement

This material is based upon work supported by the U.S. Department of Energy Office of Science under Award Number DE-FG02-04ER54738.